Quantified Modal Logic and Quine's Critique: Some Further Observations

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In "Quantified Modality and Essentialism" I pointed out that given the notion of "propositional function" modality can be extended to a quantified version. In Quine's "Three Grades of Modal Involvement" (1976) the first grade can be extended to the second (as in S5), and the logic can be extended to a quantified system, where the predicate letters of the system range over arbitrary propositional functions (functions from objects to propositions).

In this sequel I wish to discuss the issue in another setting. We won't worry about the construction of the second grade of modal involvement from the first. Let us assume the second grade of modal logic to be achieved. And we can assume the semantics for modal logic to be given as in the possible worlds semantics of my 1963a and 1965.

Now, take any non-empty set D of objects. What I have pointed out is that (inspired by the Russellian phrase 'propositional function') if we take any set of worlds to be a proposition, it is clear that a quantified version of modal logic makes sense. Let the schematic letters be replaceable by arbitrary terms for propositional functions, from the domain D to the propositions.

So what can Quine's objections to quantified modal logic amount to here?[1] It cannot be that quantified modal logic is *impossible*. It is possible. But there is the following. Ordinary predicates such as 'weighs ten kilograms', 'is six feet tall', etc., determine properties, at least

[1] That 'propositional function' might mean any function from objects to propositions is really a terminology due to Ramsey. See Ramsey (1931), p. 52.

according to intensional logic. Not only does a predicate have an actual extension: with respect to each possible world, an extension would exist also. But what is that extension? Doesn't it depend on how we 'cross identify' the individuals in each world?

Assume that such a cross identification is arbitrary. Then an ordinary predicate has no determinate extension with respect to worlds other than the actual world. Hence, no propositional function (in the sense that I have been using this term) is determined by most ordinary predicates. Thus, though quantified modal logic (using 'propositional functions') is definitely possible, it does not apply to ordinary predicates, which do not determine propositional functions.

Note that although above I have written about monadic propositional functions and predicates, the arguments obviously apply to propositional functions and predicates with any finite number of places.

The following observations should also be made:

Some predicates such as 'self-identity', 'being liquid if liquid', and so on, are obviously necessarily true of every object no matter how it is described, and Quine's grounds for skepticism about necessary properties would not apply to them. It would be possible for a semantics for modal logic to be based on the idea that predicates such as those (predicates A(x) such that (x)A(x) is necessary) are the only predicates determining essential properties. This idea can be carried out to give an interpretation of quantified modal logic. In fact, such an interpretation has actually been carried out.[2]

The notion of propositional function, contrasted with properties determined by predicates, has been presented here in terms of possible world semantics. But for the interpretation of

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[2] See Fine (1978), section II.

quantified modal logic in terms of propositional functions, there is no reason to know anything about the nature of propositions other than that they are objects closed under the truth-functions (and having a division into true and false that respects the usual truth tables for these) and the modal connectives. These need not be sets of possible worlds. The propositions can be whatever entities one likes, as long as they have the right closure properties. They can be those just stated, or they may be closed under other intensional operators such as belief. But the interpretation is genuinely arbitrary − entities called 'propositions' closed under appropriate operations, and propositional functions from some domain to these. So quantifiers can be added under very general conditions to propositional logics with intensional operators, as long as we are considering arbitrary propositional functions.

Returning to modal logic in the narrow sense, everyone knows that the methods contemplated above are not the way I myself did quantified modal logic. I did not assume a fixed domain, nor did I regard identity across possible worlds as arbitrary. And philosophically, I have defended a much more substantive theory of essential properties, and have denied that there is a genuine problem of identity across possible worlds. See my paper 1963b and *Naming and Necessity*. [3]

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